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Statistical Signal Processing for Quantum Error Mitigation

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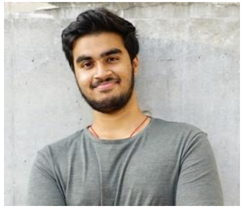




Motivation

- Quantum can provide major speedups over classical
- Quantum computers are **noisy**
  - Error rates $\geq 10^{-4}$
  - Transistor error rates $\approx 10^{-27}$
- Qubit counts growing rapidly

- Run S **shots** of a quantum circuit
  - Each shot – single measurement
  - **Expensive:** 0.1¢ – 8¢ per shot (Amazon Braket)
  - Corrupted by noise
- Extract information from noisy measurements

How do we extract as much information as possible?

Problem Formulation

Quantum algorithms on n qubits: 2^n possible outputs

BV:[1] 1 output

Grover:[2] ≥ 1 outputs

QPE:[3] Few significant outputs

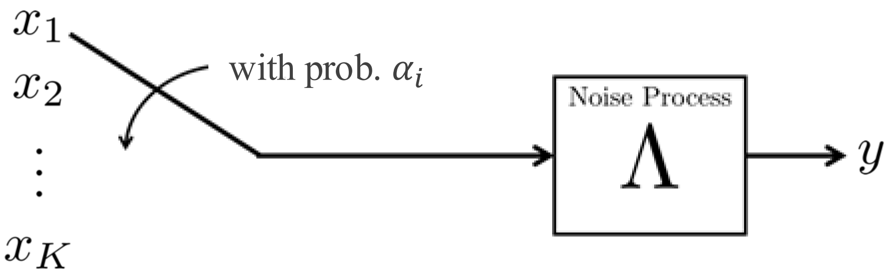
Consider algorithm, produces K **correct outputs**

Example

$$\psi = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$
$$K = 2$$

[1] Bernstein and Vazirani, 1993
[2] Grover, 1996
[3] Kitaev and Yu, 1995

Probabilistic Model



Correct output x_i selected with prob. α_i

Correct outputs chosen randomly with prob. α_i then subject to noise process

x_i: i^{th} possible solution
y: noisy data

Ideal Error Mitigation Scheme

- Handles many qubits (>100)[1]
- Does not need to observe correct outputs →
- Supports large[2] range of K
- Must be able to handle significant (>90%) depolarizing noise
  - Increasingly likely with deeper circuits[3]

Can we always observe correct outputs?

Consider $n = 200$, $\varepsilon = 0.05$, $S = 10{,}000$

10 bit flips on average

$\mathrm{P}[\text{no flips}] < 4 \times 10^{-5}$

[1] https://www.ibm.com/quantum/blog/large-scale-ftqc
[2] Montanaro, 2016
[3] Quek et. al., 2024

Existing Methods

M3 [Nation et al. 2021]

Bitstring transition probabilities

HAMMER [Tannu et al. 2022]

Considers Hamming spectrum of the shots

Q-BEEP [Stein et al. 2023]

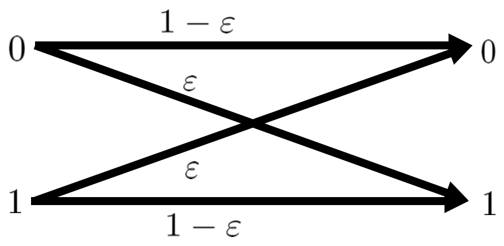
Poisson modeling over Hamming spectrum

Need to observe correct outputs

Noise Model

Bit-flip Noise

- Each bit flipped with prob. ε
- Natural model for readout error



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Depolarizing Noise

- Output drawn from uniform distribution over all 2^n bitstrings with prob. p
- Effectively captures average noise behavior in large circuits

$$\rho \rightarrow (1-p)\rho + p\pi$$

ρ: density operator
p: prob. of depolarizing
π: maximally mixed state

Estimating Correct Outputs

Based on the noise model, we have a probabilistic formulation for the error mitigation problem

S shots

K correct outputs

ε bit flip prob.

p depolarizing prob.

Maximum Likelihood Estimation

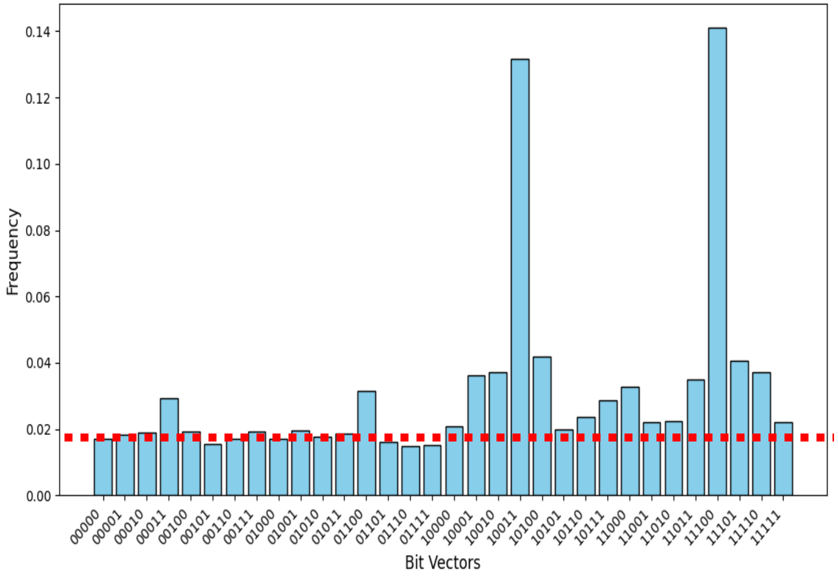
Probability of parameters θ given data Y

$$\hat{\theta} = \underset{\theta \in \Theta}{\arg\max}\, \mathcal{L}(\theta \mid Y)$$

- Classical filtering for depolarization
- Maximize likelihood function for bit flip noise model
- Nonconvex likelihood: Expectation Maximization (EM) algorithm

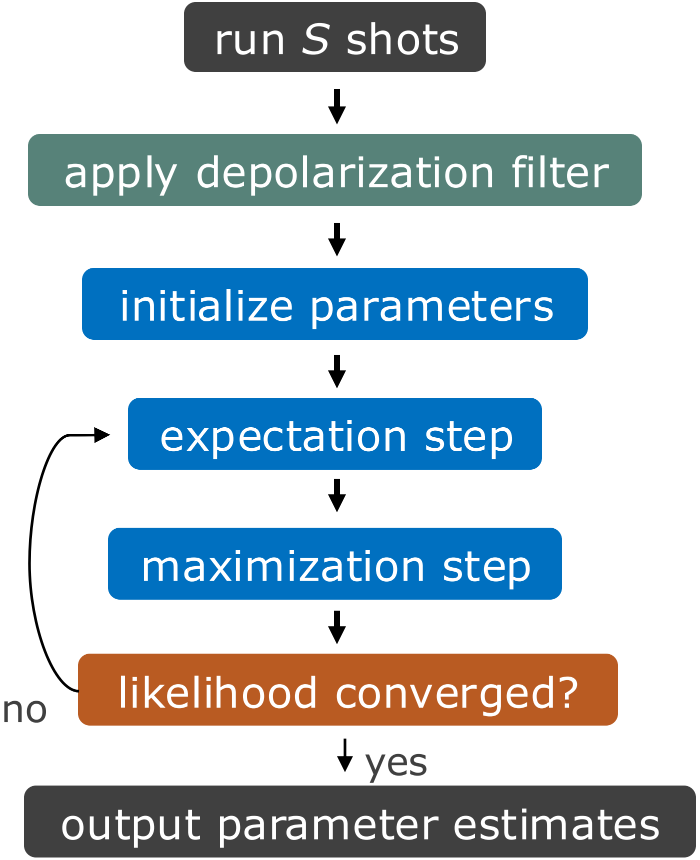
Filtering for Depolarization

- Depolarized shots distributed uniformly
  - Look at bitstring counts
  - Unlikely to have many neighbors
- Count number of neighbors
- Statistical decision threshold



Expectation Maximization Algorithm

- EM algorithm finds local maximum of likelihood function[1]
- Alternating expectation (E) and maximization (M) steps
- Estimates optimal parameters
  - No. of correct solutions
  - Correct bit strings, weights
  - Bit flip probabilities



[1] Figueiredo and Jain, 2002

Numerical Results (Simulated)

Simulated noisy quantum circuits in Qiskit using IBM machine noise model

Simulated noisy circuit data

Shots: 10,000

$K = 8$ correct outputs

BER of EM Algorithm for $K = 8$, Depth $= 800$

Number of qubits	Bit Error Rate
10	$< 1\%$
12	$< 1\%$
14	$< 1\%$

Consistently low bit error rates

Numerical Results (Hardware)

Limited testing using quantum hardware

IBM Brisbane QPU data

Shots: 16,000

Hellinger fidelity metric

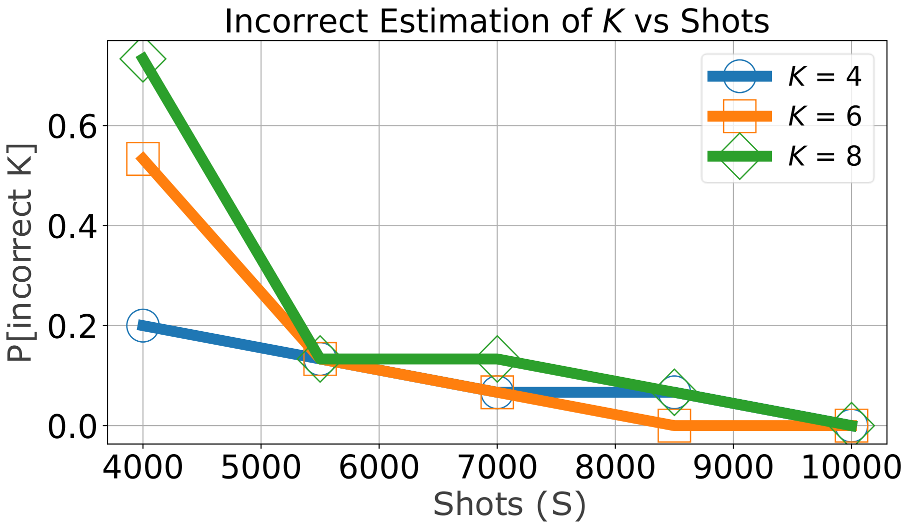
Circuit	M3	HAMMER	Q-BEEP	EM
toffoli_n3	**1.000**	0.946	0.935	**1.000**
fredkin_n3	**1.000**	0.935	0.929	**1.000**
wstate_n3	**1.000**	0.884	0.988	**1.000**
adder_n4	**1.000**	0.932	0.871	0.928
qec_en_n5	**1.000**	0.902	0.830	0.968
adder_n10	0.170	0.217	0.205	**0.732**
ghz_state_n11	0.683	**0.850**	0.453	**0.849**
bv_n14	0.034	0.104	0.024	**1.000**
qaoa_n10	0.421	**0.495**	0.363	0.023

Significant improvement on BV, adder, and GHZ circuits

Numerical Results (Shot Scaling)

Simulated noisy circuit data

$n = 14$ qubits



Larger K (correct solutions) is more difficult

Increasing shots reduces error

Conclusions

- Maximum likelihood estimation (approximate)
- Scalable for large n and significant depolarizing noise
- Strong numerical results
- Flexible probabilistic formulation
- Address most of the ideal characteristics outlined

Thanks!

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