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Statistical Signal Processing for Quantum Error Mitigation

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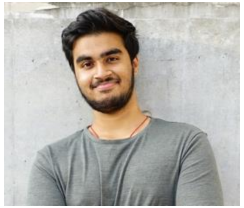



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Motivation

- Quantum can provide major speedups over classical
- Quantum computers are **noisy**
  - Error rates $\geq 10^{-4}$
  - Transistor error rates $\approx 10^{-27}$
- Qubit counts 10x per 3-4 years

- Run S **shots** of a quantum circuit
  - Each shot – single measurement
  - **Expensive:** 0.1¢ – 8¢ per shot (Amazon Braket)
  - Corrupted by noise
- Extract information from noisy measurements

How do we extract as much information as possible?

Problem Formulation

Quantum algorithms on n qubits: 2^n possible outputs

Consider algorithm, produces K **correct outputs**

Example

01110110	00010110
01010110	01010000
01111110	10110110

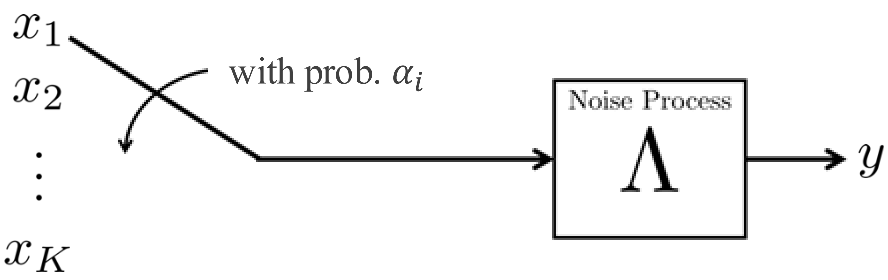
BV:[1] 1 output

Grover:[2] $\geq$ 1 outputs

QPE:[3] Few significant outputs

[1] Bernstein and Vazirani, 1993
[2] Grover, 1996
[3] Kitaev and Yu, 1995

Probabilistic Model



Correct output x_i selected with prob. α_i

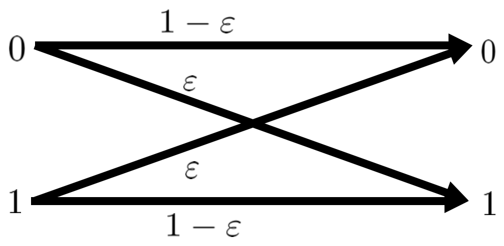
Correct outputs chosen randomly with prob. α_i then subject to noise process

x_i: i^{th} possible solution
y: noisy data

Noise Model

Bit-flip Noise

- Each bit flipped with prob. ε
- Natural model for readout error



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Depolarizing Noise

- Output drawn from uniform distribution over all 2^n bitstrings with prob. p
- Effectively captures average noise behavior in large circuits

$$\rho \to (1-p)\rho + p\pi$$

ρ: density operator
p: prob. of depolarizing
π: maximally mixed state

Existing Methods

M3 [Nation et al. 2021]

Bitstring transition probabilities

HAMMER [Tannu et al. 2022]

Considers Hamming spectrum of the shots

Q-BEEP [Stein et al. 2023]

Poisson modeling over Hamming spectrum

Need to observe correct outputs

Observing Correct Outputs

Can we always observe correct outputs?

Consider $n = 200$, $\varepsilon = 0.1$, $S = 10{,}000$

20 bit flips on average

$\mathrm{P[no\ flips]} \leq\ 10^{-9}$

Practical quantum advantage: >100 qubits

Ideal Error Mitigation Scheme

Ideal Error Mitigation Scheme

- Handles many qubits (>100)
- Does not need to observe correct outputs
- Supports large K
- Must be able to handle significant (>90%) depolarizing noise
  - Increasingly likely with deeper circuits

Algorithm Design

Estimating Correct Outputs

Based on the noise model, we have a probabilistic formulation for the error mitigation problem

Maximum Likelihood Estimation

$$\hat{\theta} = \arg\max_{\theta \in \Theta} \mathcal{L}(\theta \,|\, Y)$$

S shots

K correct outputs

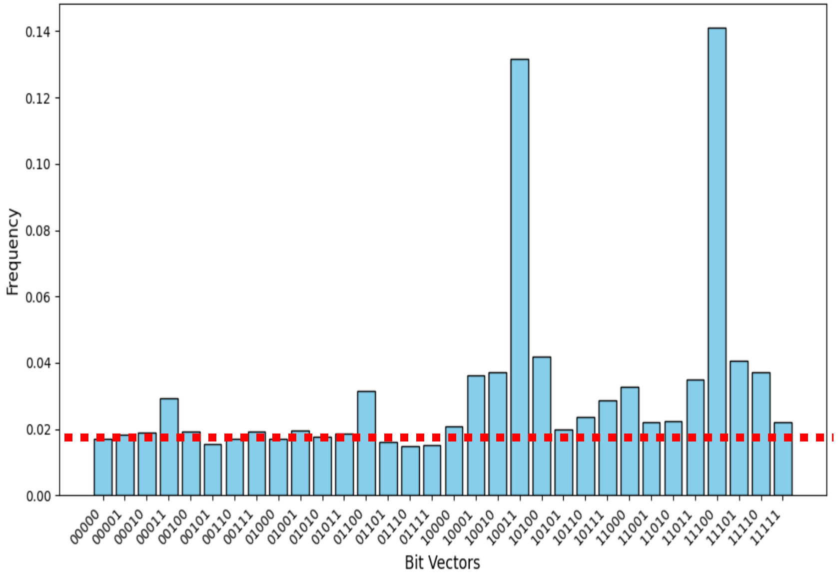
ε bit flip prob.

p depolarizing prob.

- Classical filtering for depolarizing
- Maximize likelihood function for bit flip noise model
- Nonconvex likelihood: Expectation Maximization (EM) algorithm

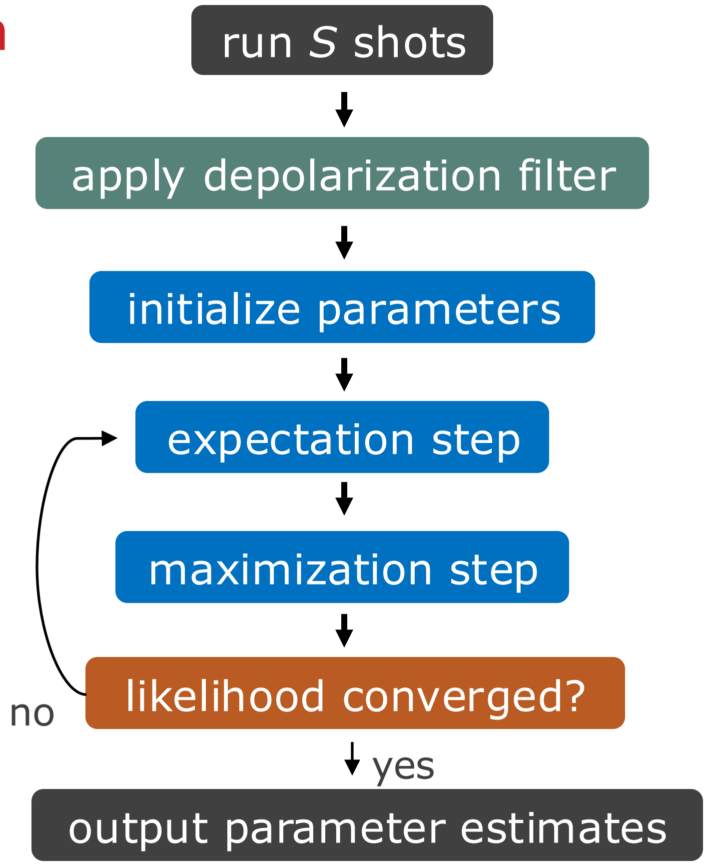
Filtering for Depolarization

- Depolarized shots distributed uniformly
  - Each bit string – approx. Poisson distribution
  - Unlikely to have many neighbors
- Count number of neighbors
- Statistical decision threshold



Expectation Maximization Algorithm

- EM algorithm finds local maximum of likelihood function[1]
- Alternating expectation (E) and maximization (M) steps
- Estimates optimal parameters
  - No. of correct solutions
  - Correct bit strings, weights
  - Bit flip probabilities



[1] Figueiredo and Jain, 2002

Numerical Results

Simulated noisy quantum circuits in Qiskit using IBM machine noise model

Simulated noisy circuit data

Shots: 10,0000

$K = 8$ correct outputs

BER of EM Algorithm for $K = 8$, Depth $= 800$

Number of qubits	**Bit Error Rate**
10	0.003
12	0.007
14	0.000

Consistently low bit error rates

Numerical Results

Limited testing using IBM quantum hardware

IBM QPU data

Shots: 16,000

Hellinger fidelity metric

Comparison Between Error Mitigation Methods

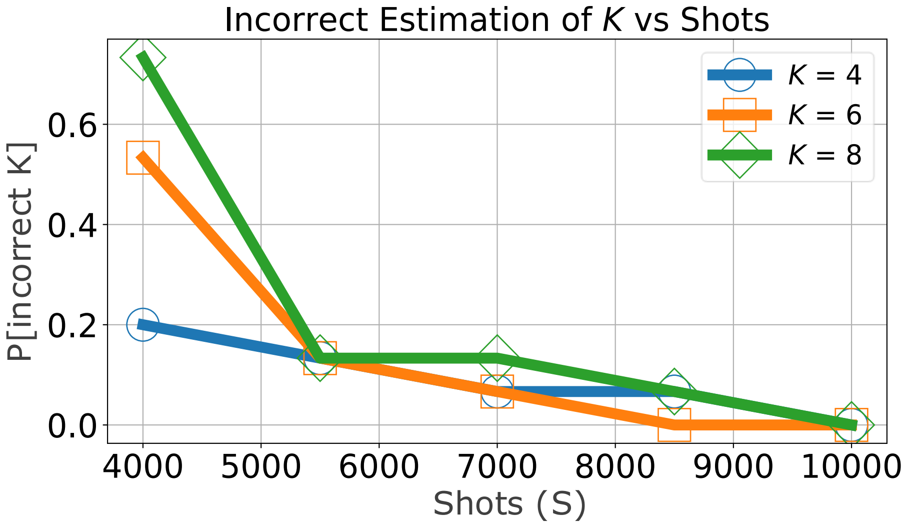
Circuit	HAMMER	M3	Q-BEEP	EM
bv_n14	0.1106	0.0219	0.0194	1.000
ghz_n11	0.8505	0.4831	0.4043	0.998
w_n3	0.8902	0.9230	0.8715	1.000
adder_n10	0.2549	0.1628	0.1497	1.000

Significant improvement on BV and adder circuits

Numerical Results

Simulated noisy circuit data

$n = 14$ qubits



Larger K (correct solutions) is more difficult

Increasing shots reduces error

Conclusions

- Maximum likelihood estimation (approximate)
- Scalable for large n and significant depolarizing noise
- Strong numerical results
- Flexible probabilistic formulation
- Address most of the ideal characteristics outlined

Future Work

- Asymmetric bit flip probabilities
- Crosstalk between qubits
- Quantum algorithms with structured output distributions
  - VQE[1]
  - QAOA[2]
- Joint QEC + QEM design

[1] Peruzzo et al., 2014
[2] Farhi et al., 2014

Thanks!

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